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The Dirac Equation with Light-Cone Data.

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ABSTRACT

The Dirac equation with light-cone data ;

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0.$$

$$u(-|x|,x) = f(x)\ ,\ x \in \mathbb{R}^3$$

is considered and explicitly solved in terms of a "Light-cone Fourier-transform" under approriate conditions on f.

1. Introduction.

We shall consider the characteristic Cauchy problem for the free Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma\cdot\nabla - m\mathbb{1})u(t,x) = 0 \tag{1.1}$$

$$u(-|x|,x) = f(x) \tag{1.2}$$

under suitable conditions on f, in four space-time dimensions.

The analogous problem for the Klein-Gordon equation was considered in [1]. The wave-equation has been considered by Riesz [2] and Strichartz [3], but in a very different way. For some general results on charateristic Cauchy-problems see Hörmander [4].

We shall show that there exist a Hilbert-space $\mathcal{H}$ of light-cone data such that (1.1) and (1.2) have a unique weak solution u(t,x), with $u(t,\cdot) \in L^2(\mathbb{R}^3, \mathbb{C}^4)$ for f in $\mathcal{H}$ and so that all "L^2 - solutions" of (1.1) have the property that $u(t-|\cdot|,\cdot)$ is in $\mathcal{H}$.

The Dirac equation (1.1) will be rewritten, as an evolution equation $i\frac{\partial u}{\partial t} = Hu$ in $\mathcal{H}$ and the generator H will be diagonalized by a "light-cone Fourier transform", which then allowe us to give an explicit formula for u(t,x) in terms of f.

2. The Dirac Equation and Associated Evolution Equations.

The Dirac equation is a hyperbolic system of linear partial differential equations which can be written as

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma\cdot\nabla - m\mathbb{1})u(t,x) = 0 \tag{2.1}$$

where $m \neq 0$ is the mass of the particle, $u: \mathbb{R}\times\mathbb{R}^3 \to \mathbb{C}^4$, γ^0 and the three components of the vector $\gamma = (\gamma^1, \gamma^2, \gamma^3)$ are 4×4 complex matrices fulfilling the anti-commutation relation

$$\gamma^\mu\gamma^\upsilon + \gamma^\upsilon\gamma^\mu = 2g^{\mu\upsilon}\,\mathbb{1} \tag{2.2}$$

where $g^{\mu\upsilon} = 0$ for $\mu \neq \upsilon$, $g^{00} = -g^{ii} = 1$, for $i = 1,2,3$, and $\mathbb{1}$ is the 4×4 unit matrix. An explicit representation of the γ^μ's is given by

$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\gamma^j = \begin{bmatrix} 0 & -\sigma^j \\ \sigma^j & 0 \end{bmatrix}$, where 1 in γ^0 is the 2×2 unit matrix and the σ^j's are the familiar Pauli-matrices given by

$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. We shall use this representation in the following.

The Dirac equation (2.1) is often considered as an evolution equation with initial data on a space-like hyperplane, consider for exsampel a $t=$ constant hyperplane and write the equation as

$i \dfrac{\partial u}{\partial t} = \gamma^0(-i\gamma\cdot\nabla + m\mathbb{1})u$, which can be considered as an evolution equation in the Hilbert-space $L^2(\mathbb{R}^3,\mathbb{C}^4)$. The generator $D = \gamma^0(-i\gamma\cdot\nabla + m\mathbb{1})$ defines a self-adjoint operator D with domain $\mathscr{D}(D) = H^1(\mathbb{R}^3,\mathbb{C}^4)$, the direct sum of the four identical Sobolev spaces $H^1(\mathbb{R}^3)$. The associated spectral representation of a solution $e^{-iDt}f$ can be written

$$u(t,x)=(2\pi)^{-3/2}\sum_{\pm}\sum_{s=1,2}\int_{\mathbb{R}^3}\frac{d^3k}{\omega_k}\, e^{\mp i\omega_k t+ik\cdot x}\, u_{\pm}(k,s)\, \hat{f}_{\pm}(k,s) \qquad (2.3)$$

for $f(\cdot) \in \mathcal{S}(\mathbb{R}^3,\mathbb{C}^4)$, the direct sum of the four identical Schwartz-spaces $\mathcal{S}(\mathbb{R}^3)$, where $u_\pm(k,s)$, $s = 1,2$, are four orthogonal eigenvectors of $\hat{D} = \gamma^0(\gamma\cdot k + m\mathbb{1})$ normalized such that $u_\pm{}^* u_\pm = \omega_k = (|k|^2 + m\mathbb{1})^{1/2}$. The "Fourier-components" $\hat{f}_\pm(\cdot,s)$ is essentially given by the ordinary Fourier-Plancherel transformation, since $L^2(\mathbb{R}^3,\mathbb{C}^4) = \bigoplus_{n=1}^{4} L^2(\mathbb{R}^3)$ it follows that the ordinary Fourier-Plancherel transform lifts in a canonical way to a unitary operator $F_0 : L^2(\mathbb{R}_x{}^3,\mathbb{C}^4) \to L^2(\mathbb{R}_k{}^3,\mathbb{C}^4)$ such that

$F_0 D F_0{}^* = \hat{D} = \gamma^0(\gamma\cdot k + m\mathbb{1})$ and $\hat{f} = F_0 f$. For $f(\cdot) \in \mathcal{S}(\mathbb{R}^3,\mathbb{C}^4)$ we have the following explicit formula

$$(F_0 f)_\pm(k,s) = \langle v_\pm(\cdot,k,s) , f(\cdot) \rangle_{L^2(\mathbb{R}_x{}^3,\mathbb{C}^4)} \tag{2.4}$$

Where

$$v_\pm(x,k,s) = (2\pi)^{-3/2} e^{ik\cdot x} u_\pm(k,s) \tag{2.5}$$

The associated Parseval's formula reads

$$\int_{\mathbb{R}^3} d^3x\ u^*(t,x)u(t,x) = \Sigma_{\pm}\Sigma_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k}\ |\ \hat{f}_{\pm}(k,s)|^2 \qquad (2.6)$$

for every $t \in \mathbb{R}$. Consider a solution $u(t,x)$ such that $\hat{f}_{\pm} \in \mathcal{S}(\mathbb{R}^3)$ and construct the current $J^{\mu} = \overline{u}\,\gamma^{\mu} u$, where $\overline{u} = u^*\gamma^0$. It follows from the Dirac equation (2.1) that J^{μ} is conserved, i.e.

$\nabla_{\mu}J^{\mu} \equiv \frac{\partial}{\partial t} J^0 + \nabla\cdot J = 0$. Intergration of this conservation equation

over the region in $\mathbb{R}\times\mathbb{R}^3$ between a $t=t_0$ hyperplane, $t_0 \in \mathbb{R}$, and the ligth-cone $\mathcal{C} = \{ (t,x) : t=t'-|x| \ , t'<t_0 \}$ together with Gauss's theorem and a decay estimate, given in [1] , gives

$$\int_{\mathbb{R}^3} d^3x\ \overline{u}(t'-|x|,x)\ (\gamma^0 + \gamma\cdot n)\ u(t'-|x|,x)$$

$$= \Sigma_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left(|\ \hat{f}_{+}(k,s)|^2 + |\ \hat{f}_{-}(k,s)|^2 \right) \qquad (2.7)$$

where $n = x|x|^{-1}$, equation (2.7) resembles a Parseval's formula.

This indicates that there might excist a Hilbert-space $\mathcal{H}$ of ligth-cone data $u(t-|\cdot|,\cdot)$ with the left-hand side of (2.7) as scalar product and such that the map $u(-|\cdot|,\cdot) \to u(t-|\cdot|,\cdot)$ defines a unitary strongly continuos one-parameter group $t \to U(t)$ on $\mathcal{H}$. This will be shown to be the case in the next section.

Let $u(0,x)$ have support in the region $|x| \leq R$ and consider $t \geq R$. It then follows from the finite propagation velocity that the support of $u(t-|x|,x)$ is in the region $|x| \leq (R+t)/2$. This means that there is a subspace $\mathcal{H}_0 \subset \mathcal{H}$ of ligth-cone data with compact support.

It follows again from the finite propergation velocity, that $U(t)\mathcal{H}_0 \subset \mathcal{H}_0$, for $t \geq 0$. This need not be the case for $t < 0$, which means that compact support of ligth-cone data, is in general, only a semi-group property.

3. The light-cone evolution equation.

Consider the Dirac equation (2.1) and make the coordinate exchange $t \to t' = t + |x|$ and $x \to x' = x$ such that a $t' =$ constant is the backward light-cone with its apex at $(t,0)$. The new coordinates will be called light-cone (LC) coordinates. In the LC coordinates the Dirac equation becomes $i(\mathbb{1} + \alpha \cdot n')\dfrac{\partial u'}{\partial t'} = (-i\alpha \cdot \nabla' + m\gamma^0)u'$ where $u'(t',x') = u(t' - |x'|, x')$, $\alpha = \gamma^0\gamma$ and $n' = x'|x'|^{-1}$, note that ∇' is associated with the light-cone $\mathcal{C} = \{ (t,x) : t = -|x| \}$ in the original coordinates and $\mathcal{C} = \{ x' \in \mathbb{R}^3 \}$ in the new coordinates, i.e. the light-cone can be identified with $\mathbb{R}^3$ which will be done in the following. We shall only use the LC coordinates from now on and can therefore drop the primes. Hence the Dirac equation becomes

$$i(\mathbb{1} + \alpha \cdot n)\frac{\partial u}{\partial t} = (-i\alpha \cdot \nabla + m\gamma^0)u \tag{3.1}$$

where $n = x|x|^{-1}$ and $\alpha = \gamma^0\gamma$.

Definition 3.1. Let $L^2 = L^2(\mho, \mathbb{C}^4)$ where the measure on $\mho$ is the Lebesgue measure on $\mathbb{R}^3$. Put

$$Q = \mathbb{1} + \alpha \cdot n \qquad \text{and} \qquad D = -i\alpha \cdot \nabla + \gamma^0 m \tag{3.2}$$

It then follows that $2^{-1}Q$ is an orthogonal projection in L^2 and that the Dirac operator D defines a self-adjoint operator in L^2 with spectrum $\sigma(D) = (-\infty,-m] \cup [m,\infty)$, $m > 0$.

The Dirac equation (3.1) can now be considered as an equation in L^2

$$iQ \frac{du}{dt}(t) = Du(t) \tag{3.3}$$

which can be written as

$$iD^{-1}Q \frac{du}{dt}(t) = u(t) \tag{3.4}$$

since $0 \notin \sigma(D)$. From equation (3.4) it follows that if $\frac{du}{dt} \in L^2$

then $u \in D^{-1}QL^2$.

Definition 3.2. Let $B = D^{-1}Q$ and $\mathcal{H}_0 = BL^2$. Futhermore put

$$\langle f,g\rangle_{\mathcal{H}_0} = \langle f,Qg\rangle_{L^2} \tag{3.5}$$

for $f,g \in \mathcal{H}_0$.

It will be proved that $\mathcal{H}_0$ is a pre-Hilbert space, but before doing that we have to mention two lemmas, both proved in [1].

Lemma 3.3. Let $S_0 = -i(\, n\cdot\nabla + \nabla\cdot n\,)$ with domain $\mathcal{D}(S_0) = C_0^\infty(\mathbb{R}^3)$ in $L^2(\mathbb{R}^3)$, then the following inequality holds

$$\| S_0 f \|_{L^2(\mathbb{R}^3)} \leq 4 \int_{\mathbb{R}^3} d^3x \; | \nabla f(x) |^2 \tag{3.6}$$

for all $f \in \mathcal{D}(S_0)$.

Lemma 3.4. The range $\mathcal{R}(S_0)$ of S_0 is dense in $L^2(\mathbb{R}^3)$.

Due to the inequality (3.6) S_0 can be extended continuously to the Sobolev space $H^1(\mathbb{R}^3)$, let this extension be denoted S , it also follows that S is symmetric in $L^2(\mathbb{R}^3)$, since S_0 is. Due to lemma 3.4. also S have dense range $\mathcal{R}(S_0)$ in $L^2(\mathbb{R}^3)$, and since S is symmetric in $L^2(\mathbb{R}^3)$ it follows that S is one-to-one, i.e. S has a densly defined inverse in $L^2(\mathbb{R}^3)$, in fact it is easy to show that

$$(S_0{}^{-1}g)(x) = \frac{i}{2r} \int_{-\infty}^{r} \rho\, g(\rho\, ,\, x|x|^{-1})\, d\rho \tag{3.7}$$

for $g \in \mathcal{R}(S_0)$.

Prpposition 3.5. $\mathcal{H}_0$ is a pre-Hilbert space.

<u>Proof.</u> The non-trivial part is to prove that if $\langle f,f\rangle_{\mathcal{H}_0} = 0$ then $f = 0$ and since $\langle f,f\rangle_{\mathcal{H}_0} = \langle f,Qf\rangle_{L^2}$ this is to prove that $Qf = 0$ implies $f = 0$ in L^2. Let $f \in \mathcal{H}_0\setminus\{0\}$. From the anti-commutation relation (2.2) it follows by a direct computasion that

$$(\alpha\cdot n)(\alpha\cdot\nabla) + (\alpha\cdot\nabla)(\alpha\cdot n) = (n\cdot\nabla + \nabla n)\mathbb{1} \tag{3.8}$$

on $C_0^\infty(\mathbb{R}^3,\mathbb{C}^4)$. In section 2 it was shown that the Dirac operator D in L^2 is self-adjoint with domain $\mathcal{D}(D) = H^1(\mathbb{R}^3,\mathbb{C}^4)$. Now the above defined operator S can be lifted in a canonical way to an operator $S\cdot\mathbb{1}$ on $\mathcal{D}(D)$, where $\mathbb{1}$ here denots the 4×4 unit matrix. Hence

$$DQ = (2-Q)D + S\cdot\mathbb{1} \tag{3.9}$$

on $\mathcal{D}(D)$, since the left-hand side and the right-hand side agree on the dense subspace $C_0^\infty(\mathbb{R}^3,\mathbb{C}^4)$. Since $0 \notin \sigma(D)$ then D have a bound invers D^{-1} defined on $\mathcal{R}(D)$, which is dense in L^2, hence D^{-1} can be extended to a continuous operator on L^2 also denoted by D^{-1}. Acting on (3.9) with D^{-1} from both left and right gives

$$QD^{-1} = D^{-1}(2-Q) + D^{-1}S\cdot\mathbb{1}D^{-1} \tag{3.10}$$

on $\mathcal{R}(D)$. By acting with Q from both left and right equation (3.10) gives

$$QB = D^{-1}S\cdot\mathbb{1}B \tag{3.11}$$

on $\{ f \in L^2 : Qf \in \mathcal{R}(D) \}$, since $Q^2 = 2Q$. Now $f = Bg \in \mathcal{H}_0\backslash\{0\}$ so it follows from equation (3.11) that $Qf = QBg = D^{-1}S\cdot\mathbb{1}Bg$ and since D^{-1} is a bounded operator and the kernel for S is trivial in $L^2(\mathbb{R}^3)$ then $D^{-1}S\cdot\mathbb{1}Bg \neq 0$, i.e. if $f \in \mathcal{H}_0\backslash\{0\}$ then $Qf \in L^2\backslash\{0\}$, or in other words if $Qf = 0$ in L^2 then $f = 0$ in $\mathcal{H}_0$.

Definition 3.6. Let $\mathcal{H}$ be the completion of $\mathcal{H}_0$.

It follows from the definition that $2^{-1/2}Q$ defines an isometry from $\mathcal{H}$ into L^2, and that B defines a bounded self-adjoint operator on $\mathcal{H}$.

Proposition 3.7. The range $\mathcal{R}(B)$ of $B : \mathcal{H} \to \mathcal{H}$ is dense in $\mathcal{H}$.

Proof. Let $\psi \in (B\mathcal{H})^{\perp}$, i.e. ψ is in the orthogonal complement of $B\mathcal{H}$, so $\langle \psi, Bf \rangle_{\mathcal{H}} = 0$ for all $f \in \mathcal{H}$. Consider $f \in \mathcal{H}_0$, then $f = Bg$ for some $g \in L^2$. Then $\langle \psi, Bf \rangle = 0$ for all $f \in \mathcal{H}$ implies that $\langle \psi, QD^{-1}QD^{-1}Qg \rangle_{L^2} = 0$ for all $g \in L^2$. Since $QD^{-1}QD^{-1}Q = (D^{-1}S \cdot \mathbb{1} D^{-1})^2 Q$ and the commutator $[Q, D^{-1}S \cdot \mathbb{1} D^{-1}] = 0$ then

$$\begin{aligned} \langle (D^{-1}S \cdot \mathbb{1} D^{-1})^2 Q\psi , g \rangle_{L^2} &= \langle QD^{-1}QD^{-1}Q\psi , g \rangle_{L^2} \\ &= \langle \psi , QD^{-1}QD^{-1}Qg \rangle_{L^2} \\ &= \langle \psi , D^{-1}QD^{-1}Qg \rangle_{L^2} = 0 \qquad (3.12) \end{aligned}$$

for all $g \in L^2$, i.e. $(D^{-1}S \cdot \mathbb{1} D^{-1})^2 Q\psi = 0$. Hence $Q\psi = 0$ in L^2 , since D^{-1} is invertibel and the kernel for S is trivial in $L^2(\mathbb{R}^3)$, i.e. $\psi = 0$ in $\mathcal{H}$ (which is proved in proposition 3.5.)

Proposition 3.7. shows that B is one-to-one since it is self-adjoint.

Definition 3.8. Put $H = B^{-1}$ with $\mathcal{D}(H) = \mathcal{R}(B)$ in $\mathcal{H}$.

Note that H is self-adjoint since B is. Equation (3.4) can then be written as

$$i \frac{du}{dt}(t) = Hu(t) \tag{3.13}$$

with the solution $u(t) = e^{-iHt}u(0)$, $u(0) \in \mathcal{D}(H)$, i.e. $U_t = e^{-iHt}$ is the time-evolution operator for light-cone data. From definition 3.8 it follows that

$$\langle f,Hf \rangle_{\mathcal{H}} = \langle f,Df \rangle_{L^2} \tag{3.14}$$

and

$$\langle Hf,Hf \rangle_{\mathcal{H}} = \langle Df,Df \rangle_{L^2} \tag{3.15}$$

for all $f \in \mathcal{D}(H)$, i.e. $\mathcal{D}(H) \subset \mathcal{D}(D) = H^1(\mathbb{R}^3,\mathbb{C}^4)$.

4. The light-cone Fourier transform.

The self-adjointness of H implies, due to the spectral theorem, that there excist a spectral family $\{ E(\lambda) \}_{\lambda\in\mathbb{R}}$ such that $H = \int_{\mathbb{R}} \lambda dE(\lambda)$. The following well-known formula allows $E(\lambda)$ to be expressed in terms of the resolvent $R(z) = (H-z)^{-1}$, for z in the resolvent set $\rho(H)$

$$\langle f , 1/2[(E(\beta)+E(\beta-0))-(E(\alpha)+E(\alpha-0))]f \rangle_{\mathcal{H}}$$

$$= \lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \, \langle f,[R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f\rangle_{\mathcal{H}} \tag{4.1}$$

In the following the rigth-hand side of equation (4.1) will be evaluated.

Let f belong to $C_0^{\infty}(\mathbb{R}^3,\mathbb{C}^4)$ and put $g = R(z)f$ for $\mathrm{Im}(z) \neq 0$.

It then follows that g belongs to $\mathcal{D}(H)$ and fulfills the equation

$$(H-z)g = f \tag{4.2}$$

which also can be written as $(B^{-1}-z)g = f$. Now applying B to this equation gives $(\mathbb{1}-zB)g = Bf$. Consider this equation in L^2, then the fact that $\mathcal{D}(H) \subset \mathcal{D}(D)$ allows us to act with D, which gives

$$(D-zQ)g = Qf \tag{4.3}$$

since $B = D^{-1}Q$.

Definition 4.1. Let V_z be defined in L^2 by

$$(V_z\varphi)(x) = e^{-iz|x|}\varphi(x) \tag{4.4}$$

on its maximal domain.

Proposition 4.2. If φ is in L^2 such that $V_z^{-1}\varphi$ is in $\mathcal{D}(D)$ then

$$(D-zQ)V_z^{-1}\varphi = V_z^{-1}(D-z)\varphi \tag{4.5}$$

Proof. Equation (4.5) follows from a straightforward computation. Rewriting equation (4.3) as $(D-zQ)V_z^{-1}V_zg = Qf$ and using proposition 4.2 gives

$$V_z^{-1}(D-z)V_z g = f \tag{4.6}$$

for $\mathrm{Im}(z) < 0$, such that V_z is bounded. Equation (4.6) can be solved for g , which gives

$$g = V_z^{-1}(D-z)^{-1}V_z Qf \tag{4.7}$$

and then

$$\langle f, R(z)f \rangle_{\mathcal{H}} = \langle V_{\bar{z}} Qf, (D-z)^{-1} V_z Qf \rangle_{L^2} \tag{4.8}$$

for $\mathrm{Im}(z) < 0$. The left-hand side of (4.8) is analytic for z not in the spectra $\sigma(H)$ of H , which then shows that the right-hand side has an analytic continuation to $\mathrm{Im}(z) > 0$.

As described in section 2 the spectral theory for the Dirac operator D in L^2 , is given by a unitary operator F_0 on L^2 , such that

$F_0 D F_0^* = \hat{D} = \alpha \cdot k + \gamma^0 m$, $k \in \mathbb{R}^3$, with eigenvalues $\pm\omega_k$. For $f \in L^2$

put for short $\hat{f} = F_0 f$. Equation (4.8) can then be written as

$$\langle f, R(z) f \rangle_{\mathcal{H}} = \langle \widehat{V_{\bar{z}} Q f} , (\hat{D} - z)^{-1} \widehat{V_z Q f} \rangle_{L^2} \tag{4.9}$$

where $(\hat{D}-z)^{-1}$ has the following explicit representation

$$(\hat{D}-z)^{-1} = \omega_k^{-1} \sum_{\pm} \sum_{s=1,2} (\pm\omega_k - z)^{-1} u_{\pm}(k,s) u_{\pm}^*(k,s) \tag{4.10}$$

and $u(\cdot,s)$ are four orthogonal eigenvectors of $\hat{D}$ normalized such

that $u_{\pm}^*(k,s) u_{\pm}(k,s) = \omega_k$ also described in section 2.

Definition 4.3. If $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$ then $f(\cdot,s)$ is defined by

$$f_{\pm}(k,s) = u_{\pm}^*(k,s) \widehat{(V_{\pm\omega_k} Q f)}(k) \tag{4.11}$$

for $s = 1,2$.

With proper interpretation formula (4.11) also can be written as

$$\hat{f}_{\pm}(k,s) = \langle u_{\pm}(\cdot,k,s), f(\cdot)\rangle_{\mathcal{H}} \tag{4.12}$$

where

$$u_{\pm}(x,k,s) = (2\pi)^{-3/2}\, e^{\pm i\omega_k |x| + ik\cdot x}\, u_{\pm}(k,s) \tag{4.13}$$

Now the rigth-hand side of equation (4.1) can be evaluated. Let $f \in C_0^{\infty}(\mathbb{R}^3, \mathbb{C}^4)$ and put

$$g_{\pm}(k,s,z) = (\, u_{\pm}^*(k,s)\widehat{(V_{\bar{z}}Qf)}(k)\,)^*(\, u_{\pm}^*(k,s)\widehat{(V_z Qf)}(k)\,) \tag{4.14}$$

then formula (4.9) becomes

$$\langle f, R(z)f\rangle_{\mathcal{H}} = \Sigma_{\pm}\Sigma_{s=1,2}\int_{\mathbb{R}^3} \frac{d^3k}{\omega_k}\quad (\pm\omega_k - z)^{-1} g_{\pm}(k,s,z) \tag{4.15}$$

Duing to the fact that $g_{\pm}$ is analytic in z and continuous in k

Fubinis theorem can be used on equation (4.15) which gives

$$\lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \, \langle f, [R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f \rangle_{\mathcal{H}}$$

$$= \sum_{\pm} \sum_{s=1,2} \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \quad \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \; h_{\pm}(k,s,\mu,\varepsilon) \tag{4.16}$$

where

$$\begin{aligned} h_{\pm}(k,s,\mu,\varepsilon) = {} & (\pm\omega_k-(\mu+i\varepsilon))^{-1} \, g_{\pm}(k,s,\mu+i\varepsilon) \\ & -(\pm\omega_k-(\mu-i\varepsilon))^{-1} \, g_{\pm}(k,s,\mu-i\varepsilon) \end{aligned} \tag{4.17}$$

Since $g_{\pm}$ is analytic in z Taylor's formula can be used and together

with Lebesgue's dominated convergence theorem equation (4.16)

becomes

$$\lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \, \langle f, [R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f \rangle_{\mathcal{H}}$$

$$= \sum_{\pm} \sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \; \pi^{-1} \lim_{\varepsilon \searrow 0} \int_{\alpha}^{\beta} d\mu \; \Psi_{\pm}(k,s,\mu,\varepsilon) \frac{\varepsilon}{(\mu\mp\omega_k)^2+\varepsilon^2} \tag{4.18}$$

where

$$\Psi_{\pm}(k,s,\mu,\varepsilon) = g_{\pm}(k,s,\mu) - g_{\pm}'(k,s,\mu)(\mu\mp\omega_k)$$

$$- \varepsilon(1/2[\mathrm{Re}(g_{\pm}''(\xi_{\pm}))+\mathrm{Im}(g_{\pm}''(\eta_{\pm}))](\pm\omega_k-(\mu-i\varepsilon))$$

$$- 1/2[\mathrm{Re}(g_{\pm}''(\xi_{\pm}'))+\mathrm{Im}(g_{\pm}''(\eta_{\pm}'))](\pm\omega_k-(\mu-i\varepsilon))) \tag{4.19}$$

here $g_{\pm}'$ denote the partial derivative of $g_{\pm}$, $g_{\pm}''$ the second partial derivative of $g_{\pm}$ in the z-variable and $0 \le \xi_{\pm},\xi_{\pm}',\eta_{\pm},\eta_{\pm}' \le \varepsilon$. Finaly since $f_{\varepsilon}(y) = \pi^{-1}\varepsilon(y^2+\varepsilon^2)^{-1}$ is a delta family (as $\varepsilon\searrow 0$) equation (4.18) gives

$$\lim_{\varepsilon\searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\mu \, \langle f,[R(\mu+i\varepsilon)-R(\mu-i\varepsilon)]f\rangle_{\mathcal{H}}$$

$$= \sum_{\pm}\sum_{s=1,2} \int_{\alpha<\pm\omega_k<\beta} \frac{d^3k}{\omega_k} \; \Psi\pm(k,s,\pm\omega_k,0) \tag{4.20}$$

i.e.

$$\lim_{\varepsilon \searrow 0} \frac{1}{2\pi i} \int_{\alpha}^{\beta} d\bar{\mu} \, \langle f, [R(\bar{\mu}+i\varepsilon) - R(\mu - i\varepsilon)] f \rangle_{\mathcal{H}}$$

$$= \begin{cases} \sum_{s=1,2} \int_{\alpha<\omega_k<\beta} \frac{d^3k}{\omega_k} \mid \hat{f}_+(k,s) \mid^2 & , m<\alpha<\beta \\[2ex] \sum_{s=1,2} \int_{-\beta<\omega_k<-\alpha} \frac{d^3k}{\omega_k} \mid \hat{f}_-(k,s) \mid^2 & , \alpha<\beta<-m \\[2ex] \sum_{s=1} \int_{\omega_k<\beta} \frac{d^3k}{\omega_k} \mid \hat{f}_+(k,s) \mid^2 & \\ + \sum_{s=1,2} \int_{\omega_k<-\alpha} \frac{d^3k}{\omega_k} \mid \hat{f}_-(k,s) \mid^2 & , \alpha<-m, \ \beta>m \\[2ex] 0 & , \text{otherwise} \end{cases} \qquad (4.21)$$

duing to the fact that $\Psi_{\pm}(k,s,\pm\omega_k,0) = g_{\pm}(k,s,\pm\omega_k) = \left| \hat{f}_{\pm}(k,s) \right|^2$.
From equation (4.1) and (4.17) it then follows that

$$\langle f,[E(\beta)-E(\alpha)]f\rangle_{\mathcal{H}}$$

$$= \begin{cases} \sum_{s=1,2} \int_{\alpha<\omega_k<\beta} \frac{d^3k}{\omega_k} \left| \hat{f}_+(k,s) \right|^2 & , m<\alpha<\beta \\ \sum_{s=1,2} \int_{-\beta<\omega_k<-\alpha} \frac{d^3k}{\omega_k} \left| \hat{f}_-(k,s) \right|^2 & , \alpha<\beta<-m \\ \sum_{s=1,2} \int_{\omega_k<\beta} \frac{d^3k}{\omega_k} \left| \hat{f}_+(k,s) \right|^2 + \sum_{s=1,2} \int_{\omega_k<-\alpha} \frac{d^3k}{\omega_k} \left| \hat{f}_-(k,s) \right|^2 & , \alpha<-m,\ \beta>m \\ 0 & , \text{otherwise} \end{cases} \tag{4.22}$$

Letting $\alpha \to -\infty$ and $\beta \to \infty$ formular (4.23) gives

$$\|f\|^2_{\mathcal{H}} = \sum_{\pm}\sum_{s=1,2} \int_{\mathbb{R}^3} \frac{d^3k}{\omega_k} \left| \hat{f}_{\pm}(k,s) \right|^2 \tag{4.24}$$

for $f \in C_0^\infty(\mathbb{R}^3, \mathbb{C}^4)$. Now the map defined in definition 4.3 can be extended by continuity to an isometry.

Definition4.4. Let $\hat{\mathcal{H}} = \hat{\mathcal{H}}_+ \oplus \hat{\mathcal{H}}_-$, $\hat{\mathcal{H}}_\pm = L^2(\mathfrak{M}_\pm, \mathbb{C}^2)$ where $\mathfrak{M}_\pm$ are the mass-hyperboloids and the measure is $\frac{d^3k}{\omega_k}$. Define

$F : \mathcal{H} \to \hat{\mathcal{H}}$ by $Ff = (\hat{f}_+, \hat{f}_-)$.

It follows from formula (2.7) that the map $F : \mathcal{H} \to \hat{\mathcal{H}}$ in fact is onto, i.e. F is unitary.

Theorem 4.5. The unitary map $F : \mathcal{H} \to \hat{\mathcal{H}}$, called the "light-cone Fourier transform", diagonalizes H , i.e.

$$FHf = (\omega_k \hat{f}_+ , -\omega_k \hat{f}_-) \tag{4.25}$$

for $f \in \mathcal{D}(H)$.

Proof. Let $f \in C_0^\infty(\mathbb{R}^3,\mathbb{C}^4)$ then

$$\begin{aligned} \widehat{Hf}_\pm(k,s) &= \langle u_\pm(\cdot,k,s) , Hf(\cdot) \rangle_{\mathcal{H}} \\ &= \langle Du_\pm(\cdot,k,s) , f(\cdot) \rangle_{L^2} \\ &= \pm\omega_k \langle u(\cdot,k,s) , Qf(\cdot) \rangle_{L^2} \\ &= \pm\omega_k \hat{f}_\pm(k,s) \end{aligned} \tag{4.26}$$

where $Du_\pm$ is understood in weak sence. Since H is closed and F is continuous, formula (4.25) holds for all $f \in \mathcal{D}(H)$.

The light-cone Fourier transform defines a kind of "duality" between the light-cone and the mass-hyperdoloids.

Recall that formula (2.3) gives rise to a unitary map from $\hat{\mathcal{H}}$ onto $L^2(\mathbb{R}^3,\mathbb{C}^4)$ of t = constant data, i.e. there is a one-to-one correspondance between weak solutions $u(t,x)$ of the Dirac equation (2.1) with $u(t,\cdot) \in L^2(\mathbb{R}^3,\mathbb{C}^4)$ and the solution $e^{-iHt}f$, $f \in \mathcal{H}$, of equation (3.13).

5. Conclusion.

We have shown that the characteristic Cauchy problem for the Dirac equation

$$(i\gamma^0 \frac{\partial}{\partial t} + i\gamma \cdot \nabla - m\mathbb{1})u(t,x) = 0$$

$$u(-|x|,x) = f(x) \quad , \quad x \in \mathbb{R}^3 \tag{5.1}$$

has a unique weak solution with $u(t,\cdot) \in L^2(\mathbb{R}^3,\mathbb{C}^4)$ for all $f \in \mathcal{H}$ and that all "L^2-solutions" are obtained in this way.

The Dirac equation was written as an evolution equation for light-cone data

$$i \frac{du}{dt} = Hu \tag{5.2}$$

and the spectral theory for H was developed including a "light-cone Fourier transform" which diagonalized H .

I want to aknowledge Department of mathematics, University of Copenhagen, Denmark where this letter is done and Department of mathematics, University of California at Santa Barbara where the final form has taken place. Especialy I want to thank Lars-Erik Lundberg Department of mathematics, University of Copenhagen, Denmark for suggesting the problem and for giving me lots of advice.

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Af: Jens Jæger.

105/85 "THE FREQUENCY DEPENDENCE OF THE SPECIFIC HEAT AF THE GLASS REANSITION".
Af: Tage Christensen.

"A SIMPLE MODEL AF AC HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.
Contributions to the Third International Conference on the Structure of Non - Crystalline Materials held in Grenoble July 1985.

106/85 "QUANTUM THEORY OF EXTENDED PARTICLES".
Af: Bent Sørensen.

107/85 "EN MYG GØR INGEN EPIDEMI",
- flodblindhed som eksempel på matematisk modellering af et epidemiologisk problem.
Projektrapport af: Per Hedegård Andersen, Lars Boye, CarstenHolst Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Jesper Larsen.

108/85 "APPLICATIONS AND MODELLING IN THE MATEMATICS CURRICULUM" - state and trends -
Af: Mogens Niss.

109/85 "COX I STUDIETIDEN" - Cox's regressionsmodel anvendt på studenteroplysninger fra RUC.
Projektrapport af: Mikael Wennerberg Johansen, Poul Katler og Torben J. Andreasen.
Vejleder: Jørgen Larsen.

110/85 "PLANNING FOR SECURITY".
Af: Bent Sørensen

111/85 "JORDEN RUNDT PÅ FLADE KORT".
Projektrapport af: Birgit Andresen, Beatriz Quinones og Jimmy Staal.
Vejleder: Mogens Niss.

112/85 "VIDENSKABELIGGØRELSE AF DANSK TEKNOLOGISK INNOVATION FREM TIL 1950 - BELYST VED EKSEMPLER".
Projektrapport af: Erik Odgaard Gade, Hans Hedal, Frank C. Ludvigsen, Annette Post Nielsen og Finn Physant.
Vejleder: Claus Bryld og Bent C. Jørgensen.

113/85 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS 11".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

114/85 "ANVENDELSE AF GRAFISKE METODER TIL ANALYSE AF KONTIGENSTABELLER".
Projektrapport af: Lone Biillmann, Ole R. Jensen og Anne-Lise von Moos.
Vejleder: Jørgen Larsen.

115/85 "MATEMATIKKENS UDVIKLING OP TIL RENÆSSANCEN".
Af: Mogens Niss.

116/85 "A PHENOMENOLOGICAL MODEL FOR THE MEYER-NELDEL RULE".
Af: Jeppe C. Dyre.

117/85 "KRAFT & FJERNVARMEOPTIMERING"
Af: Jacob Mørch Pedersen.
Vejleder: Bent Sørensen

118/85 "TILFÆLDIGHEDEN OG NØDVENDIGHEDEN IFØLGE PEIRCE OG FYSIKKEN".
Af: Peder Voetmann Christiansen

119/86 "DET ER GANSKE VIST - - EUKLIDS FEMTE POSTULAT KUNNE NOK SKABE RØRE I ANDEDAMMEN".
Af: Iben Maj Christiansen
Vejleder: Mogens Niss.

120/86 "ET ANTAL STATISTISKE STANDARDMODELLER".
Af: Jørgen Larsen

121/86 "SIMULATION I KONTINUERT TID".
Af: Peder Voetmann Christiansen.

122/86 "ON THE MECHANISM OF GLASS IONIC CONDUCTIVITY".
Af: Jeppe C. Dyre.

123/86 "GYMNASIEFYSIKKEN OG DEN STORE VERDEN".
Fysiklærerforeningen, IMFUFA, RUC.

124/86 "OPGAVESAMLING I MATEMATIK".
Samtlige opgaver stillet i tiden 1974-jan. 1986.

125/86 "UVBY,β - systemet - en effektiv fotometrisk spektralklassifikation af B-,A- og F-stjerner".
Projektrapport af: Birger Lundgren.

126/86 "OM UDVIKLINGEN AF DEN SPECIELLE RELATIVITETSTEORI".
Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

127/86 "GALOIS' BIDRAG TIL UDVIKLINGEN AF DEN ABSTRAKTE ALGEBRA".
Projektrapport af: Pernille Sand, Heine Larsen & Lars Frandsen.
Vejleder: Mogens Niss.

128/86 "SMÅKRYB" - om ikke-standard analyse.
Projektrapport af: Niels Jørgensen & Mikael Klintorp.
Vejleder: Jeppe Dyre.

129/86 "PHYSICS IN SOCIETY"
Lecture Notes 1983 (1986)
Af: Bent Sørensen

130/86 "Studies in Wind Power"
Af: Bent Sørensen

131/86 "FYSIK OG SAMFUND" - Et integreret fysik/historieprojekt om naturanskuelsens historiske udvikling og dens samfundsmæssige betingethed.
Projektrapport af: Jakob Heckscher, Søren Brønd, Andy Wierød.
Vejledere: Jens Høyrup, Jørgen Vogelius, Jens Højgaard Jensen.

132/86 "FYSIK OG DANNELSE"
Projektrapport af: Søren Brønd, Andy Wierød.
Vejledere: Karin Beyer, Jørgen Vogelius.

133/86 "CHERNOBYL ACCIDENT: ASSESSING THE DATA.
ENERGY SERIES NO. 15.
AF: Bent Sørensen.

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen, Petr Visčor

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSESTEORETISKE FORUDSÆTNINGER"
MASTEMATIKSPECIALE: Claus Larsen
Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens første og andet møde med græsk filosofi"
Projektrapport af Frank Colding Ludvigsen
Vejledere: Historie: Ib Thiersen
Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE FASTE STOFFER" - Resume af licentiatafhandling
Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International Workshop on Teaching Nonlinear Phenomena at Universities and Schools, "Chaos in Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

139/87 "Machbarkeit nichtbeherrschbarer Technik durch Fortschritte in der Erkennbarkeit der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR - ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircan Approach

Paper presented at the Symposium on the Foundations of Modern Physics The Copenhagen Interpretation 60 Years after the Como Lecture. Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

145/87 "AIMS AND SCOPE OF APPLICATIONS AND MODELLING IN MATHEMATICS CURRICULA"

Manuscript of a plenary lecture delivered at ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

146/87 "BESTEMMELSE AF BULKRESISTIVITETEN I SILICIUM"

- en ny frekvensbaseret målemetode.

Fysikspeciale af Jan Vedde

Vejledere: Niels Boye Olsen & Petr Viščor

147/87 "Rapport om BIS på NAT-BAS"

redigeret af: Mogens Brun Heefelt

148/87 "Naturvidenskabsundervisning med Samfundsperspektiv"

af: Peter Colding-Jørgensen DLH
Albert Chr. Paulsen

149/87 "In-Situ Measurements of the density of amorphous germanium prepared in ultra high vacuum"

by: Petr Viščor

150/87 "Structure and the Existence of the first sharp diffraction peak in amorphous germanium prepared in UHV and measured in-situ"

by: Petr Viščor

151/87 "DYNAMISK PROGRAMMERING"

Matematikprojekt af:
Birgit Andresen, Keld Nielsen og Jimmy Staal

Vejleder: Mogens Niss

152/87 "PSEUDO-DIFFERENTIAL PROJECTIONS AND THE TOPOLOGY OF CERTAIN SPACES OF ELLIPTIC BOUNDARY VALUE PROBLEMS"

by: Bernhelm Booss-Bavnbek
Krzysztof P. Wojciechowski

153/88 "HALVLEDERTEKNOLOGIENS UDVIKLING MELLEM MILITÆRE OG CIVILE KRÆFTER"

Et eksempel på humanistisk teknologihistorie
Historiespeciale

Af: Hans Hedal

Vejleder: Ib Thiersen

154/88 "MASTER EQUATION APPROACH TO VISCOUS LIQUIDS AND THE GLASS TRANSITION"

By: Jeppe Dyre

155/88 "A NOTE ON THE ACTION OF THE POISSON SOLUTION OPERATOR TO THE DIRICHLET PROBLEM FOR A FORMALLY SELFADJOINT DIFFERENTIAL OPERATOR"

by: Michael Pedersen

156/88 "THE RANDOM FREE ENERGY BARRIER MODEL FOR AC CONDUCTION IN DISORDERED SOLIDS"

by: Jeppe C. Dyre

157/88 " STABILIZATION OF PARTIAL DIFFERENTIAL EQUATIONS BY FINITE DIMENSIONAL BOUNDARY FEEDBACK CONTROL: A pseudo-differential approach."

by: Michael Pedersen

158/88 "UNIFIED FORMALISM FOR EXCESS CURRENT NOISE IN RANDOM WALK MODELS"

by: Jeppe Dyre

159/88 "STUDIES IN SOLAR ENERGY"

by: Bent Sørensen

160/88 "LOOP GROUPS AND INSTANTONS IN DIMENSION TWO"

by: Jens Gravesen

161/88 "PSEUDO-DIFFERENTIAL PERTURBATIONS AND STABILIZATION OF DISTRIBUTED PARAMETER SYSTEMS:

Dirichlet feedback control problems"

by: Michael Pedersen

162/88 "PIGER & FYSIK - OG MEGET MERE"

AF: Karin Beyer, Sussanne Blegaa, Birthe Olsen,
Jette Reich , Mette Vedelsby

163/88 "EN MATEMATISK MODEL TIL BESTEMMELSE AF PERMEABILITETEN FOR BLOD-NETHINDE-BARRIEREN"

Af: Finn Langberg, Michael Jarden, Lars Frellesen

Vejleder: Jesper Larsen

164/88 "Vurdering af matematisk teknologi
Technology Assessment
Technikfolgenabschätzung"

Af: Bernhelm Booss-Bavnbek, Glen Pate med
Martin Bohle-Carbonell og Jens Højgaard Jensen

165/88 "COMPLEX STRUCTURES IN THE NASH-MOSER CATEGORY"

by: Jens Gravesen

166/88 "Grundbegreber i Sandsynligheds-
regningen"

Af: Jørgen Larsen

167a/88 "BASISSTATISTIK 1. Diskrete modeller"

Af: Jørgen Larsen

167b/88 "BASISSTATISTIK 2. Kontinuerte
modeller"

Af: Jørgen Larsen

168/88 "OVERFLADEN AF PLANETEN MARS"
Laboratorie-simulering og MARS-analoger
undersøgt ved Mössbauerspektroskopi.

Fysikspeciale af:

Birger Lundgren

Vejleder: Jens Martin Knudsen
Fys.Lab./HCØ

169/88 "CHARLES S. PEIRCE: MURSTEN OG MØRTEL
TIL EN METAFYSIK."

Fem artikler fra tidsskriftet "The Monist"
1891-93.

Introduktion og oversættelse:

Peder Voetmann Christiansen

170/88 "OPGAVESAMLING I MATEMATIK"

Samtlige opgaver stillet i tiden
1974 - juni 1988